Mass Operator and the Gauge Field Theory

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We introduce the Mass Operator ($\hat{m} = i\hbar\frac{\partial}{\partial\tau}$) in comparing the correspondence between mass-energy equation and invariant space-time interval in Special Relativity with the correspondence between operators and classical physical quantities in Quantum Mechanics, where the quantity τ is the proper time of a single particle. The commutation rejection of the mass operator is drawn out in the derivation of the Dirac equation. And the formula of the plane wave of a freely-propagating particle has been generalized to be $exp[\frac{-i}{\hbar}(-Et + \vec{P}\cdot\vec{x} + m\cdot\tau)]$. Furthermore, we investigate the local gauge transformation properties of the Dirac equation after we introduced the mass operator into the theory. It turns out that in this case the mass term of non-Ablian gauge fields can be introduced in theory and none of auxiliary fields is needed.
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I. INTRODUCTION

In the past several decades of years, the Special Relativity and the Quantum Mechanics have been the groundwork of the modern physics. They are two of few well defined physical theories which have been established on the axiomatized architecture.

As we all know that the observer-invariant space-time interval in the Special Relativity is

$$ds^2 = -dt^2 + d\vec{x}^2. \tag{1}$$

In parallel, the mass-energy equation in the Special Relativity is in the familiar form

$$-m^2 = -E^2 + \vec{P}^2. \tag{2}$$

On the other hand, the correspondence between operators and classical physical quantities in Quantum Mechanics may be denoted as

$$E \longrightarrow i\hbar\frac{\partial}{\partial t} = \hat{E}; \tag{3}$$

$$P_i \longrightarrow i\hbar\frac{\partial}{\partial x^i} = \hat{P}_i. \tag{4}$$

Comparing Eq.(2) with Eqs.(3-4), we may believe that there must be an operator corresponded to the mass of a single particle. Now we consider a particle moving in a rest inertial frame. Suppose this particle is emitted (event 1) at the origin of the coordinates $\vec{x} = 0$ when $t = 0$ and arrives (event 2) at time t at coordinate $\vec{x}$. The space-time interval of these two events may be written as

$$ds^2 = -t^2 + \vec{x}^2. \tag{5}$$

Furthermore, we can chose the proper time of the particle in event 1 to be zero, and in event 2 to be τ. Then we have

$$-\tau^2 = -t^2 + \vec{x}^2. \tag{6}$$

In analog to the correspondence of (3-4), from Eq.(2) and Eq.(6), it is natural that

$$\hat{m} = i\hbar\frac{\partial}{\partial\tau}, \tag{7}$$

where the operator $\hat{m}$ is just the mass operator which we want to introduce into our theory.

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II. DIRAC EQUATION OF A FREE ELECTRON

After having introduced the mass operator, we can investigate its commutation rejection. The best way to achieve it is to recover the classical Dirac equation. In the spirit of Dirac, the relativistic equation of a freely-propagating particle should be constituted by the linear operators, and must be one order. Therefore, the relativistic equation of a free electron with a mass operator may be written as

$$[i\hbar\frac{\partial}{\partial t}-c\alpha^j(i\hbar\frac{\partial}{\partial x^j})-c^2\beta(i\hbar\frac{\partial}{\partial \tau_e})]\psi=0. \tag{8}$$

Where τ_e is the proper time of the electron. Multiplying $[i\hbar\frac{\partial}{\partial t}+c\alpha^j(i\hbar\frac{\partial}{\partial x^j})+c^2\beta(i\hbar\frac{\partial}{\partial \tau_e})]$ on the both sides of Eq.(8), we have

$$(\hat{E}+c\alpha^j\hat{P}_j+c^2\beta\hat{m})(\hat{E}-c\alpha^j\hat{P}_j-c^2\beta\hat{m})\psi=0. \tag{9}$$

By simple expansion of the right hand side of Eq.(9), we further have

$$[\hat{E}^2-c^2\alpha^i\alpha^j\hat{P}_i\hat{P}_j+c\alpha^i(\hat{P}_i\hat{E}-\hat{E}\hat{P}_i)+\beta c^2(\hat{m}\hat{E}-\hat{E}\hat{m})-c^3(\alpha^i\beta\hat{P}_i\hat{m}+\beta\alpha^i\hat{m}\hat{P}_i)-\beta^2c^4\hat{m}^2]\psi=0 \tag{10}$$

Considering the existence of the mass operator $\hat{m}$, we should naturally generalize the formula of the plane wave of free particles to be

$$\psi=u(P_\mu,m)\cdot e^{\frac{-i}{\hbar}(-Et+\vec{P}\cdot\vec{x}+m\cdot\tau)}. \tag{11}$$

To recover the classical formula of Dirac equation, the commutation rejection of the mass operator as follow would be necessary,

$$\hat{m}\hat{E}-\hat{E}\hat{m}=0; \tag{12}$$
$$\hat{m}\hat{P}_i-\hat{P}_i\hat{m}=0; \tag{13}$$

and

$$(\alpha^1)^2=(\alpha^2)^2=(\alpha^3)^2=\beta^2=1; \tag{14}$$
$$\alpha^i\alpha^j+\alpha^j\alpha^i=0; \tag{15}$$
$$\alpha^i\beta+\beta\alpha^i=0. \tag{16}$$

Finally, using these commutation relations in (10) yields the Klein-Gordon equation

$$[\hat{E}^2-c^2\hat{P}^2-c^4{\hat{m}_e}^2]\psi=0 \tag{17}$$

Using the expansion (11) on the (17), the requirement from the Special Relativity (2) is naturally met. Furthermore, according to the Dirac's definition of

$$\gamma^i=-\beta\alpha^i, \tag{18}$$
$$\gamma^0=\beta, \tag{19}$$

here we have chosen the nature units ($c=\hbar=1$), then Eq.(8) may be written as

$$(i\gamma^\mu\partial_\mu-\hat{m}_e)\psi=0. \tag{20}$$

The corresponding lagrangian can also be taken as

$$\mathcal{L}_0=\bar{\psi}(i\gamma^\mu\partial_\mu-\hat{m}_e)\psi. \tag{21}$$

III. ELECTROMAGNETIC GAUGE FIELD THEORY

Electromagnetic gauge field can be introduce by considering a local $U(1)$ gauge transformation as follows

$$\psi\longrightarrow e^{i\alpha_p(x,\tau_p)}\psi, \tag{22}$$

here τ_p is the proper time of the photon, then the random phase factor $\alpha_p(x,\tau_p)$ satisfies the gauge of electromagnetic field

$$A_\mu \longrightarrow A_\mu - \frac{1}{e}\partial_\mu \alpha_p(x,\tau_p). \tag{23}$$

The whole lagrangian invariant under the transformation (22) can be written as

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu\partial_\mu - \hat{m}_e)\psi - e\bar{\psi}\gamma^\mu A_\mu \psi + \hat{m}_p A_\mu \hat{m}_p A^\mu. \tag{24}$$

The proper time of the photon is the constant in its propagating, so $\hat{m}_p A_\mu = 0$, the mass term of the Electromagnetic gauge field is vanished.

IV. NON-ABLIAN GAUGE FIELD THEORY

In convenient, we here discuss the $SU(2)$ local gauge transformation

$$\psi \longrightarrow e^{i\frac{\tau^i \alpha_i(x,\lambda_i)}{2}}\psi, \tag{25}$$

now the λ_i is denoted as the proper time of the propagator of the non-Ablian gauge field A^i_μ and τ^i is the generator of $SU(2)$ group.

We write down the whole lagrangian of the non-Ablian gauge theory directly

$$\mathcal{L} = -\frac{1}{4}F^i_{\mu\nu}F^{i\mu\nu} + \bar{\psi}(i\gamma^\mu\partial_\mu - \hat{m}_e)\psi - g\bar{\psi}\gamma^\mu \frac{\vec{\tau}\cdot\vec{A_\mu}}{2}\psi + \hat{m}_i A^i_\mu \hat{m}_i A^{i\mu}. \tag{26}$$

The definition of $F^i_{\mu\nu}$ is given by

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g\epsilon^i_{jk}A^j_\mu A^k_\nu. \tag{27}$$

All quantities have been naturally defined to be consistent with their original formulaes. To prove (26) is invariant under a local gauge transformation of (25), we can decompose the phase factor $\alpha_i(x,\lambda_i)$ to be

$$\alpha_i(x,\lambda_i) = \alpha_i(x) + \alpha_i(\lambda_i), \tag{28}$$

which can always be realized because that: to any a physical operation of gauge transformation, as long as we keep consistence on the ultimate phasic difference of the wave function ψ, it would always be reasonable to achieve it in two steps, one is pure of space-time, and the another is pure of the proper time of all propagators. To the first step,

$$\psi \longrightarrow e^{i\frac{\tau^i \alpha_i(x)}{2}}\psi. \tag{29}$$

We have

$$\begin{aligned}
\hat{m}_i A'^i_\mu &= \hat{m}_i(A^i_\mu + \epsilon^{ijk}\alpha_j A_{k\mu} - \frac{1}{g}\partial_\mu\alpha^i) \\
&= \hat{m}_i A^i_\mu + \epsilon^{ijk}\hat{m}_i(\alpha_j A_{k\mu}) - \frac{1}{g}\hat{m}_i(\partial_\mu\alpha^i) \\
&= \hat{m}_i A^i_\mu
\end{aligned} \tag{30}$$

Where $\hat{m}_i A_{k\mu} = 0$ is for $i \neq k$, and $\hat{m}_i(\partial_\mu\alpha^i) = 0$ is for $\alpha^i = \alpha^i(x)$. In the meantime, the leading three terms on the right-hand side of (26) is invariant under the transformation of (29) because they are in the same formulaes with their original ones. Thus the whole lagrangian (26) is invariant under the local transformation (29). To the second step,

$$\psi \longrightarrow e^{i\frac{\tau^i \alpha_i(\lambda_i)}{2}}\psi, \tag{31}$$

For here $\alpha_i(\lambda_i)$ is irrespective to the coordinates of space-time, then the transformation (31) is equivalent to a global gauge transformation to the leading three terms on the right-hand side of (26), so they are still invariant under this transformation. To the last term on the right-hand side of (26), we also have $\hat{m}_i(\partial_\mu\alpha^i) = 0$ for $\partial_\mu\alpha^i = \partial_\mu\alpha^i(\lambda_i)$. On the other hand, $\alpha_j = \alpha_j(\lambda_j)$ and $i \neq j$, so $\hat{m}_i\alpha_j = 0$. In conclusion, the whole lagrangian of non-Ablian gauge field theory (26) is invariant under the local transformation of (25).

V. CONCLUSIONS

The standard Higgs mechanism and its Higgs boson, as a striking prediction of the electroweak theory, still have not been verified at the present time. The attempt to combining the Higgs scalar together with the gravitational-field theory is also made in an alternative scenario[1]. This is a possibility, but such a gravity theory still have problems. Meanwhile, as another important prediction of the Standard Model, there are still existing the serious difficulties in applying the concept of spontaneous-breaking vacuum energy to modern cosmology [2, 3, 4, 5]. In this Letter we want to demonstrate a novel scenario of gauge field theory through introducing the mass operator $\hat{m}$. In addition, the formula of the plane wave of a freely-propagating particle is also generalized to be $exp[\frac{-i}{\hbar}(-Et + \vec{P} \cdot \vec{x} + m \cdot \tau)]$. The observational effects from this generalization can be figured out by investigating a massive field in further consideration. Furthermore, according to the eigen-equation of the mass operator

$$i\hbar \frac{\partial}{\partial \tau} \psi = m\psi, \tag{32}$$

mass is just the eigenvalue of an eigen-equation. The mass spectrum of elemental particles may also be comprehended along this way.

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